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Dynamos driven by poloidal flow exist

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Abstract. We have discovered a class of dynamos driven by purely poloidal fluid motion, thereby demonstrating that toroidal motion is not essential for dynamo action and that there is no counterpart to the anti-dynamo theorem that purely toroidal motions cannot generate magnetic fields. The fully three-dimensional (3D) dynamo action of these models results from the pushing and twisting of magnetic field lines by helical motion in a manner akin to the α^2 mechanism of mean-field electrodynamics. No dynamo with dipole symmetry was found for an axisymmetric distribution of helicity; indeed, some azimuthal variation in helicity is required. Among the suite of dynamos that we have investigated, the poloidal flow dynamo with the smallest critical magnetic Reynolds number is very nearly the dynamo with the most nonaxisymmetric distribution of helicity.

Introduction

Dynamo action in the Sun and the Earth is sustained by the motion of electrically conducting fluid. In a kinematic dynamo analysis, such as that discussed here, one investigates the types of fluid motion which sustain dynamo action by solving the magnetic induction equation,

$$\partial_t \mathbf{B} = R_m \nabla \times (\mathbf{u} \times \mathbf{B}) + \nabla^2 \mathbf{B}, \qquad (1)$$

where $\mathbf{B}$ denotes the magnetic field and $\mathbf{u}$ is the dimensionless fluid velocity. The magnetic Reynolds number, R_m, gives a measure of the effectiveness by which fluid motion acts to amplify the magnetic field compared to diffusive decay due to electrical resistance.

Early work on the dynamo resulted in some negative results, the so-called anti-dynamo theorems; the most famous being that due to *Cowling* [1934], who showed that fluid motion cannot generate axisymmetric magnetic fields, a result which holds for time-dependent fields and time-dependent compressible fluids [*Hide and Palmer*, 1982]. And recently it has been shown that fluid motion cannot sustain purely toroidal magnetic fields [*Kaiser, Schmitt and Busse*, 1994]. In addition to theorems concerning the magnetic field are theorems concerning the nature of the velocity field. *Elsasser* [1946] and *Bullard and Gellman* [1954] showed that poloidal motion is essential for dynamo action, i.e. that toroidal motion alone is insufficient to sustain a magnetic field, some radial poloidal motion is necessary. Other analyses have bounded the minimum amount of radial motion necessary for dynamo action [*Busse*, 1975], and have bounded the minimum size of the magnetic Reynolds number [*Backus*, 1958; *Childress*, 1969; *Proctor*, 1977]. Our discovery of dynamo action sustained by purely poloidal motion demonstrates that there is no counterpart to the toroidal flow anti-dynamo theorem.

Method

We consider a conducting fluid sphere surrounded by a stationary electrical insulator. Equation (1) is then solved by the Bullard-Gellman method [*Bullard and Gellman*, 1954]: discretizing the equation by expanding both the velocity field and magnetic field in terms of

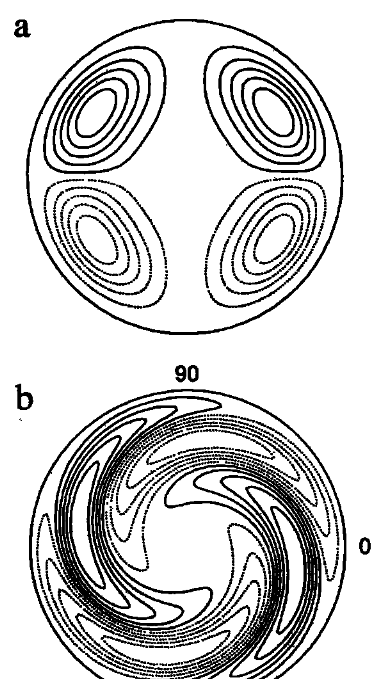

Figure 1. Poloidal fluid velocity components: (a) meridional section showing streamlines of meridional circulation, s_2^0, (b) equatorial section showing streamlines of convective motion, $\mathrm{s}_2^{2s} + \mathrm{s}_2^{2c}$.

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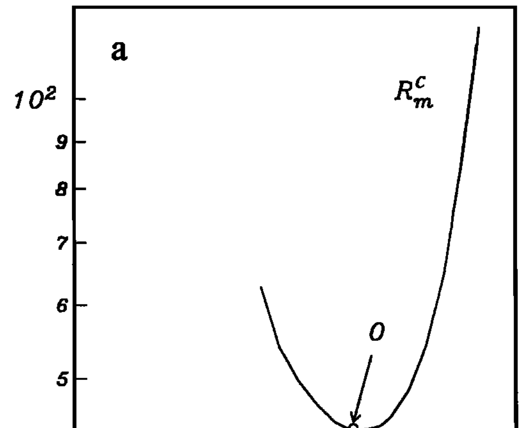

Figure 2. (a) Critical magnetic Reynolds number, R^c_m, as a function of the ratio of meridional motion to convective motion, ϵ_1/ϵ_2. Model $\mathcal{O}$ is the dynamo with smallest R^c_m. (b) The average helicity, $\langle h\rangle^2$, and the nonaxisymmetric helicity, $\langle h\rangle^2_{NA}$, as a function of ϵ_1/ϵ_2. Note that model $\mathcal{O}$ is very nearly the model of maximal $\langle h\rangle^2_{NA}$.

toroidal and poloidal vector harmonics and radial grid points. For a prescribed velocity field, $\mathbf{u}$, the induction equation is reduced to an algebraic eigenvalue problem. Standard numerical techniques are used to solve for the magnetic field, $\mathbf{B}$, and the critical magnetic Reynolds number, R^c_m, at which dynamo action occurs. R^c_m is defined so that $\langle \mathbf{u}\rangle = 1$, where $\langle\cdots\rangle$ denotes a volumetric RMS average.

Kumar and Roberts [1975] found numerically convergent magnetic fields sustained by a flow of the form

$$\mathbf{u} = \epsilon_0 \mathbf{t}^0_1 + \epsilon_1 \mathbf{s}^0_2 + \epsilon_2\left[\mathbf{s}^{2s}_2 + \mathbf{s}^{2c}_2\right], \qquad (2)$$

where $\{\epsilon_0, \epsilon_1, \epsilon_2\}$ are adjustable parameters. Toroidal motion, $\mathbf{t}^0_1$, is differential rotation, whilst the poloidal component, $\mathbf{s}^0_2$, is meridional motion which has been found to promote steady solutions in $\alpha\omega$-dynamos [*Roberts*, 1972]. The sectoral poloidal harmonics, $\mathbf{s}^{2s}_2$ and $\mathbf{s}^{2c}_2$, contribute convective overturning with multi-cellular motion three-deep along the radius. In Fig. 1 we show the form of $\mathbf{s}^0_2$ and $\mathbf{s}^{2s}_2 + \mathbf{s}^{2c}_2$.

Previous studies using (2) addressed the Braginsky limit, $\epsilon_0 \to \infty$, and thus were concerned with dynamo action dominated by differential rotation [*Kumar and Roberts*, 1975]. In this study we fix ϵ_0 equal to zero, and therefore restrict ourselves to purely poloidal motion; we explore dynamo action by varying the relative proportion of meridional and convective motions as measured by the ratio ϵ_1/ϵ_2. We search for magnetic fields with dipole symmetry, i.e. antisymmetric upon reflection through the equatorial plane, as is geophysically and heliophysically relevant.

Results and Discussion

A suite of steady poloidal flow dynamos were found. All are dominated by strong meridional motion, with negative ϵ_1, meaning that the sense of the meridional motion is one of upwelling along the equator and downwelling along the geographic poles. The critical magnetic Reynolds number, R^c_m, as a function of ϵ_1/ϵ_2, is shown in Fig. 2a. Dynamo efficiency is optimum for model $\mathcal{O}$, where the magnetic Reynolds number attains a minimum, $R^c_m \simeq 44$ for $\epsilon_1/\epsilon_2 = 6$.

The magnetic field of $\mathcal{O}$ is shown in Fig. 3, where it is revealed that meridional motion promotes the sustenance of poloidal axisymmetric fields; for the optimum dynamo 80% of the magnetic energy is axisymmetric and 78% is poloidal. The surface field of $\mathcal{O}$ is extremely simple, Fig. 3b, consisting of two flux patches concentrated near the geographic poles, the result of strong meridional motion, which sweeps poloidal magnetic field lines towards the poles and concentrates them by fluid downwelling. The magnetic field is fairly complicated in the fluid interior, Fig. 3c,e,f, but the azimuthally averaged field, Fig. 3a,d, is large scale and does not strongly reflect the underlying multi-cellular convective flow. The numerical convergence of $\mathcal{O}$ is verified by comparing magnetic spectra for two different truncations of harmonic expansion; Fig. 4.

In mean-field electrodynamics [*Steenbeck, Krause and Rädler*, 1966], α regeneration of magnetic field results from the average induction sustained by short length-scale helical motion. Our dynamo models are fully 3D, there has been no averaging and they are not mean-field dynamos. However, it is instructive to consider the dynamo action of our models in terms of helicity, $h = \mathbf{u}\cdot(\nabla\times\mathbf{u})$, and its regenerative effects [*Parker*, 1979]. Helicity arises from the cross products of pairs of velocity harmonics. The convective terms $\mathbf{s}^{2s}_2$ and $\mathbf{s}^{2c}_2$ generate only axisymmetric helicity, $\langle h\rangle_{AS}$, whilst the

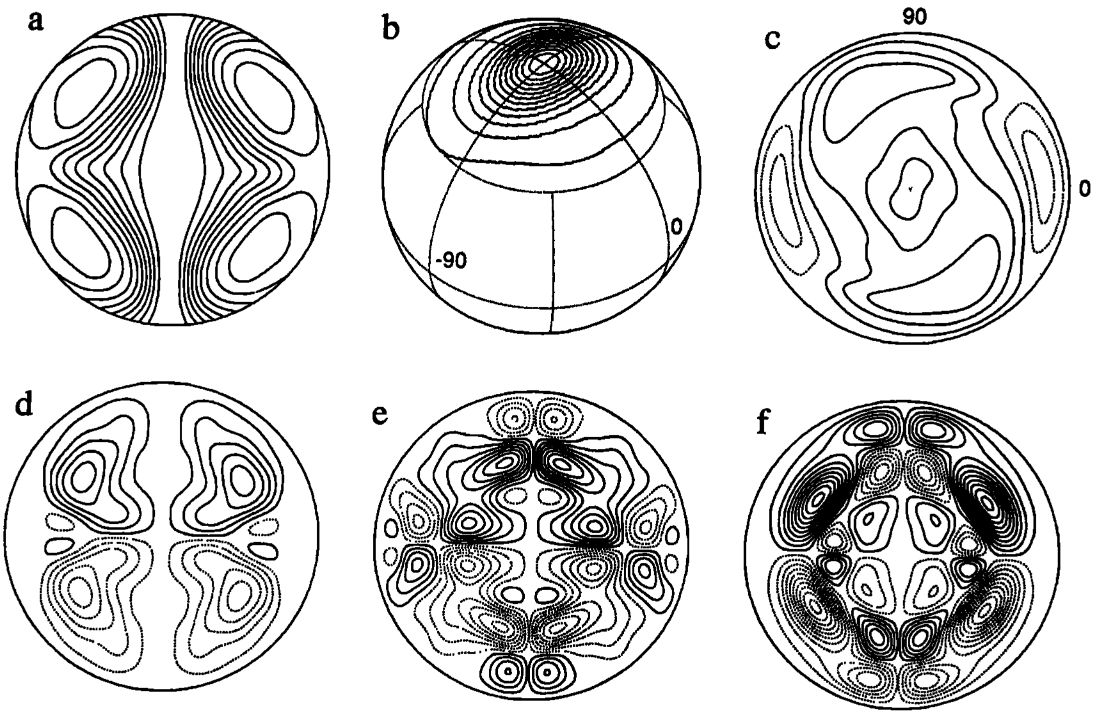

Figure 3. The magnetic field for model $\mathcal{O}$: (a) meridional section showing azimuthally averaged magnetic field lines, (b) satellite view showing contours of B_r, (c) equatorial section showing contours of B_θ, (d) meridional section showing azimuthally average contours of B_ϕ, (e) meridional section showing contours of B_ϕ at longitude $\phi = 0$, (f) meridional section showing contours of B_ϕ at longitude $\phi = 90$.

meridional circulation $\mathbf{s}_2^0$ combines with each convective term to give nonaxisymmetric helicity, $\langle h \rangle_{NA}$, varying in azimuth as $\sin 2\phi$, see Fig. 5.

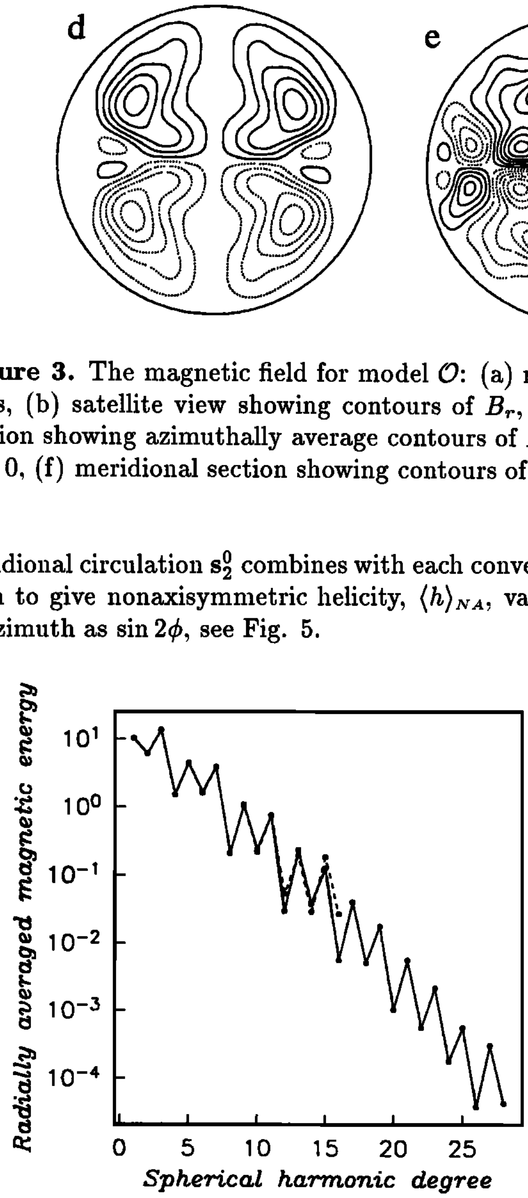

Figure 4. Magnetic energy of $\mathcal{O}$ as a function of spherical harmonic degree. The dashed (solid) line represents the spectrum of the magnetic field calculated with a spherical harmonic expansion truncated at degree 16 (28). The energy decreases with increasing harmonic degree, and since the models are adequately converged, the inclusion of higher degree terms, those above degree 16, has little effect on the overall solutions.

No dipole-symmetric dynamo was found with $\epsilon_1 = 0$, the case of axisymmetric helicity, $\langle h \rangle_{NA} = 0$, and no dipole-symmetric dynamo is sustained for $\epsilon_2 = 0$, the case of no helicity, $\langle h \rangle = 0$. Interestingly, the most efficient of our dynamo models, that with the smallest R_m^c, model $\mathcal{O}$, is the dynamo with very nearly the largest $\langle h \rangle_{NA}$, see Fig. 1b. This result indicates that the nonaxisymmetric distribution of helicity is an important constituent in dynamos with dipole symmetry. That the spatial arrangement of helicity is an important factor in dynamo efficiency has been reported previously [*Love and Gubbins*, 1996].

In mean-field theory, α is proportional to the mean helicity and for an α^2 dynamo one might expect the most efficient dynamo would be the one with the largest $\langle h \rangle_{AS}$. However these dynamos are macroscopic, and although they do not possess any toroidal differential rotation and appear to operate through a mechanism somewhat akin to the α^2 mechanism, they are fully 3D and thus helicity cannot be simply related to α generation. Axisymmetric helicity distorts field lines uniformly around the axis, which subsequently leads to cancellation rather than reinforcement of the preexisting field. In these dynamos cancellation is reduced by the meridional circulation, which imparts an azimuthal variation to the helicity, $\langle h \rangle_{NA}$, and an azimuthal variation in magnetic regeneration as required by Cowling's theorem. It is this 3D induction, not the helicity itself, which, when spatially averaged, gives a net α-effect.

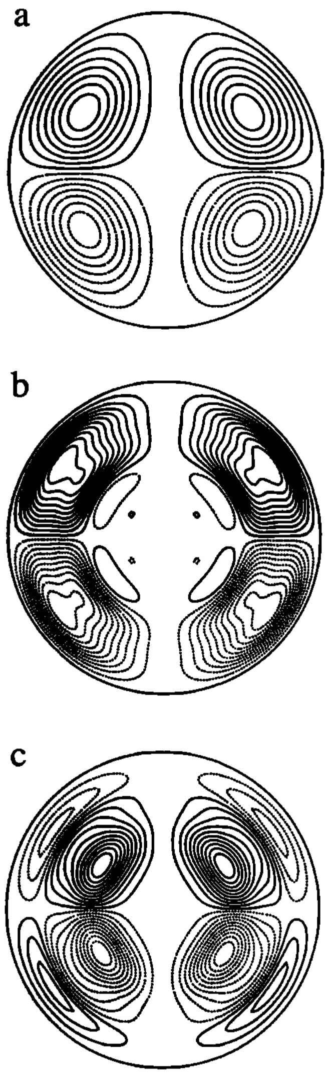

Figure 5. Contours of helicity for model $\mathcal{O}$ in meridional sections: (a) azimuthally averaged helicity, (b) helicity at longitude $\phi = 0$, (b) helicity at longitude $\phi = 90$.

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