Problem of the Love-Gannon relation between the asymmetric disturbance field and $\text{Dst}$

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[1] Love and Gannon (2009) discovered that statistically, over a fifty year period the difference in the dawn and dusk disturbance-field $H$ component at low latitudes (hourly averaged) is linearly proportional to $\text{Dst}$. If the difference is designated by $\delta_{DD}$ in units of nT/RE, then the Love-Gannon (L-G) relation is $\delta_{DD} = -0.2 \text{Dst}$. At any time departures from the relation can be large. Nonetheless, the relation is evident for all values of $\text{Dst}$ and persists throughout magnetic storms, both the main phase and the recovery phase. The Love-Gannon discovery presents a problem to current understanding of the relation between the causes of $\delta_{DD}$ and $\text{Dst}$ because the dawn dusk asymmetry in the disturbance field is presumably governed by a long-established magnetosphere-ionosphere coupling theory which predicts a characteristic time scale (the shielding time) of less than an hour whereas the characteristic time scale for $\text{Dst}$ (the ring current decay time) is more like ten hours. Thus, without forcing both time scales toward each other to the limits of their ranges, a linear proportionality between $\delta_{DD}$ and $\text{Dst}$ cannot be derived from the current understanding of the causes of the asymmetry and the ring current. This conclusion is the paper’s main contribution. In addition, we attempt to get around the conflict of time scales by looking at other possibilities for generating $\delta_{DD}$ that depend directly on the ring current. The most promising of these is the possibility that the ring current decay mechanism creates a quasi-permanent, local-time modification of the ring current compared to what it would be in the absence of the decay mechanism and that this modification causes a field-aligned current that closes through the ionosphere and generates the asymmetry $\delta_{DD}$. This idea has the virtue of coupling the asymmetry directly to the ring current and of accounting for the persistence of the L-G proportionality through the recovery phase of magnetic storms.


1. The Paper’s Purpose

[2] This paper reports on an attempt to find a physical cause for a regularity recently discovered by Love and Gannon [2009] in the local-time asymmetry in the magnetic field measured by ground observatories at low latitude. The regularity is this: the dawn-dusk asymmetry in the disturbance field is on average proportional to $\text{Dst}$. That so simple a result was revealed half a century after continuous recording of the $\text{Dst}$ index began is surprising, reasons for which we consider in section 4. But more important than this surprise is the property of asymmetries like this to cause a force between the geomagnetic dipole and some current-carrying plasma in the ionosphere, magnetosphere, or solar wind and thus to be diagnostic of the existence of an important dynamical process. We give examples of such processes that are known in section 4, but the late discovery of the Love and Gannon asymmetry has not yet been matched by a discovery of the dynamical processes to which it is diagnostic. As we report here, we have managed to eliminate some possibilities and are left with one that we have been unable to eliminate. We begin by discussing the phenomenology of the Love-Gannon asymmetry.

2. The Love-Gannon Relation: What Is It?

[3] Love and Gannon [2009] demonstrate that the dawn-dusk asymmetry in the low-latitude geomagnetic disturbance field is statistically proportional to $\text{Dst}$. The proportionality is well marked in a 50-year span of data. These data scatter around the line of strict proportionality (as shown in Figure 2), but whereas the scatter seems random, the linearity that Love and Gannon discovered is well defined.

[4] Figure 1 shows the Love-Gannon regularity (Figure 1, left) and an example of a variation from it (Figure 1, right).
The quantity plotted in Figure 1 (left) is the disturbance in the low-latitude $H$ component normalized to $Dst$ ($\delta H/Dst$). The blue, saw-tooth line traces the 24 one-hour local time values of the normalized disturbance averaged over 50 years (1958–2007) [from Love and Gannon, 2009]. The smooth red line is a two-harmonic fit to the blue line. It is nearly a circle whose center is offset from the origin in the dusk direction by about 0.2 units. This is a visual demonstration of the Love-Gannon relation

$$\delta_{DD} = -0.2 \ Dst$$  

(1)

where $\delta_{DD}$ is the symbol that we will use to designate the dawn-dusk asymmetry that is statistically correlated with $Dst$. At any time the dawn-dusk asymmetry may not and often does not obey equation (1), as Figure 1 (right) shows. If it did, however, we could say that $\delta_{DD}$ is simply half the difference between the disturbance fields at dawn and dusk. The “one-half” factor gives $\delta_{DD}$ the units $nT/R_E$ (where $R_E$ denotes the radius of the Earth) since dawn and dusk are separated by the diameter of the Earth ($2R_E$). (This statement takes account of the convention of geomagneticians to project values to the equator by dividing by the cosine of the latitude of the observing station.) Also if it were usually the case that the dawn-dusk asymmetry is related to $Dst$ by equation (1), the Love-Gannon relation would have been discovered much earlier and have another name.

Figure 1 (right) shows the local time variation of $\delta H$ at the $Dst$-peak of a geomagnetic storm [from Love and Gannon, 2010]. The inner circle is the zero from which to

The correlation between $Dst$ and the dawn-dusk asymmetry in the low latitude $H$ component of the geomagnetic disturbance field from 1985 to 2009. The ordinate labeled (Dawn – Dusk)/2 is commensurate with $\delta_{DD}$. The solid line is a plot of the Love-Gannon relation (equation (1)). Dashed lines are least squares fits to the data shown. Colors labeled by percentages refer to the sample cadence, for example, the blue, 10% points mark every tenth data point and the green, 0.1% points every one-thousandth data point.
measure the disturbance value at any local time (negative outward, positive inward) using the scale given by the dotted circles. (The radial lines mark fiducial longitudes which are not relevant here.) The labeled red and blue dots show the local time positions of stations that provided the data from which the trace was constructed. For example, the disturbance at dawn is $-300$ nT and at dusk $-650$ nT. Thus, in this instance the value of the dawn-dusk asymmetry is $(650 - 300)/2 = 175$ nT/Re, which happens to be about $-0.4$ Dst. The example illustrates by how much instantaneous values of the dawn-dusk asymmetry can deviate from the L-G relation, which in this case gives $\delta_{DD} = 84$ nT/Re.

The coefficient $-0.2$ in the Love-Gannon relation is the lowest order rounding off of a least squares fit to the data (see Figure 2). Rounding off de-emphasizes the exact value of the coefficient and emphasizes the fact of the proportionality itself. That is, the interesting thing is that statistically $\delta_{DD}$ scales linearly with $\text{Dst}$, whatever the scaling factor happens to be (in fact about $-0.2$).

Figure 2 shows the dawn-dusk asymmetry (one-half the dawn-dusk difference in the $H$ component) for data from 1-h windows centered on dawn and dusk using USGS data covering the years 1985 to 2009. Figure 2 (left) gives one-minute data and Figure 2 (right) shows one-hour averages of the data. The straight line in the figures shows the L-G relation (equation (1)), which has a slope of $-0.2$. Least squares fits to these data (dashed lines) give slopes a little different than $-0.2$, but as mentioned precision is not the issue; it is the revealed fact that statistically the dawn-dusk asymmetry tends to be about 20% of $\text{Dst}$. Scatter away from the line in all directions is large, giving each figure the appearance (ignoring color) of a splash from a moving liquid. Note that the dawn-dusk differences are about as often negative as positive relative to the $L$-G line and by comparable amounts. Thus, the L-G relation holds only statistically, but for reasons stated in Section 4, it is nonetheless remarkable.

3. The L-G Relation and the Kyoto SYM-H and ASY-H Indices

The Kyoto World Data Center compiles indices that quantify the symmetrical and non-symmetrical parts of the low-latitude disturbance of the $H$ field, designated SYM-H and ASY-H, respectively. Figure 3 gives an example of how the L-G relation is manifested in these data. The data are presented in a format that can be compared with Figure 2. Here, however, ASY-H is always positive, being the absolute value of the local-time range of $H$; (i.e., the difference between maximum $H$ and minimum $H$, regardless of where in local time these values occur), whereas the dawn-dusk asymmetry in Figure 2 is one-half the signed, dawn and dusk difference of the $H$ component. Thus, one should fold the points in Figure 2 about the L-G line to make it look like Figure 3. This difference in the appearance of the figures merely reflects different definitions of variables and not a difference in phenomena. However, the difference in the scales on the ordinates in the two figures (maximum 200 nT in Figure 2 versus 500 nT in Figure 3) does reveal something about the phenomena not inferable from Figure 2 alone, and this is that the biggest contributions to the Kyoto ASY-H come from local-time differences that are not in the dawn-dusk meridians to which Figure 2 is restricted. Examples of strong, non-dawn-dusk contributions are mentioned below.

The point of the figure is that the L-G relation is evident as a marked correlation between ASY-H and SYM-H even during a major magnetic storm.

We have plotted ASY-H versus SYM-H for other storms. In nearly all cases the correlation has two branches, a high ASY-H branch and a low ASY-H branch (the Halloween storm of Figure 3 is an exception). The high ASY-H branch corresponds to the main phase and the low ASY-H branch to the recovery phase. In general the low ASY-H branch lies closer to the L-G line, although the L-G trend appears also in the high ASY-H branch. That the main phase branch usually lies above the L-G line is not surprising because this is when strong, non-dawn-dusk asymmetries happen, caused by substorms, sawtooth events, and a strong noon-midnight asymmetry related to the region 1 current system (see below). Evidently during the main phase of most storms, these contributions to ASY-H usually surpass the dawn-dusk asymmetry described by the Love-Gannon relation. The closer adherence of recovery-phase points to the L-G relation suggests that the relation itself
might have something to do with ring current decay, which is a suggestion to which we return later.

[10] The information to extract from the typical storm is that the L-G relation is imposed on the correlation between ASY-H and SYM-H throughout the storm, but during the main phase the ASY-H component is often accompanied by stronger asymmetries from processes other than the (unknown) L-G process.

[11] A plot (Figure 4) similar to Figure 2 was obtained by Weygand and McPherron [2006] based on a statistical analysis of 162 storms aligned essentially on the peak depressions in Sym-H* (here the asterisk indicates a correction for solar wind pressure). The contours in Figure 4 show the relative frequency in which the symmetric and asymmetric components of the 162-storm composite disturbance field simultaneously have the values specified by the ordinate and abscissa. The ordinate in Figure 4 should have the units nT/2RE that is the full difference between maximum and minimum. Figure 4 shows main phase data and Figure 4 (right) shows recovery phase data. The authors divided their data into times for which IMF Bz < 0 and IMF Bz > 0. Figure 4 shows negative IMF Bz plots, but the positive IMF Bz plots are similar. The contours display a linear alignment, which in both panels is close to Asym-H/Sym-H* = 7/10 or in our units ASY*-H/Sym-H* = 3.5/10. This is bigger than the 2/10 ratio of the L-G relation, but the different statistical treatments might account for the difference in the ratios. (For example, the W-M Asym-H includes all local times as does Kyoto ASY-H, thus one expects a higher ratio. One sees this in Figure 3a, for which a simple linear fit gives a slope of −0.3.) The important point is that this study, too, finds a dominant linear relation between the symmetrical and asymmetrical components of the low-latitude H-disturbance field during both main and recovery phases.

4. Why the L-G Relation Is Interesting

[12] Figures 1–4 imply that the L-G relation is a system-wide response of the magnetosphere to stimulation by the solar wind. That a system-wide regularity this pronounced has escaped remark for so long is surprising. It stayed out of sight until Weygand and McPherron [2006] noticed it and Love and Gannon [2009] quantified it. One reason for this might be that Sugiura and Chapman did not find it in the first major study of the relation between Dst and the asymmetry in the low-latitude disturbance field [Sugiura and Chapman, 1960]. In this widely referenced work, Sugiura and Chapman superposed Dst and a measure of the disturbance field asymmetry (the first harmonic of the local-time profile, call it 1stLT) of 74 magnetic storms, using the storm sudden commencement to align the time profiles. The storm-time profiles of Dst and 1stLT look different. The latter peaks very soon after the sudden commencement, then decays slowly to zero by the end of the recovery phase; whereas the peak depression in Dst occurs while 1stLT is decaying. Thus the ratio 1stLT/Dst is not constant as in the L-G relation but drops monotonically from greater than one to essentially zero through the storm.

[13] Part of the Weygand and McPherron [2006] project was to understand why Sugiura and Chapman [1960] failed to find a marked trend toward constancy in the 1stLT-to-Dst ratio, like the plots in Figures 3 and 4. Weygand and McPherron were able to reproduce the Sugiura-Chapman result by moving the zeros of both the Sym-H* and Asym-H

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**Figure 4.** Plots similar to Figure 2 compiled by Weygand and McPherron [2006] showing a linear relation between a Dst proxy (Sym-H*, the asterisk signifying “pressure corrected”) and an index of the local time range in H based on a particular set of low-latitude stations.
axes by about 20 nT. This answer, however, jumps over the question: did Sugiura and Chapman have the right zeros? That is, should we regard Sugiura-Chapman to be the default result to aim for? Figure 3a suggests that there are at least two types of correlation between the symmetric and asymmetric disturbance fields. There is a fast one that acts like the Sugiura-Chapman correlation in that asymmetry is big when Dst is small and vice versa. Possibly in superposing 74 storms this correlation swamps the smaller amplitude, slow L-G correlation.

[14] Even in the one-hour averages that Love and Gannon used, non-L-G behavior is evident as we noted in discussing Figure 2. The hourly averaged local disturbance sometimes differs by 50 nT from the L-G linear fit. Evidently the physics behind the L-G relation competes with other mechanisms that can produce big asymmetries that last an hour or more. One such mechanism is compression of the magnetosphere by the solar wind which causes a noon-midnight aligned asymmetry, stronger $H$ at noon, which in units of nT/R$_E$ happens to be numerically close to the solar wind dynamic pressure in nPa [Siscoe, 1966]. Figure 5 (left) shows an example in which compression during the initial phase of a storm in November 2004 caused an approximately 20 nT/R$_E$ asymmetry of the stated type, which is about the value of the concurrent solar wind dynamic pressure. Since no theory predicts it, the tight coupling between $\delta_{DD}$ and Dst expressed by the L-G relation presents an interesting puzzle.

[15] Another reason that the L-G relation has remained hidden is that it is unexpected; no theory predicts it, hence, no one has been looking for it. Instead, common understanding predicts its absence. The dawn-dusk asymmetry is normally discussed in the context of a partial ring current closing through the ionosphere via field-aligned currents whereas Dst is associated with a symmetric ring current (other contributions exist, for instance from the tail current system, but are not relevant to the argument) with no necessary, systematic connection to the partial ring current or to the ionosphere. Since no theory predicts it, the tight coupling between $\delta_{DD}$ and Dst expressed by the L-G relation presents an interesting puzzle.

[16] We proceed under the assumption that there is a system-wide process by which the solar wind produces the L-G relation, that this process operates essentially all of the time, but that it is frequently swamped by other processes that also produce low-latitude H-field asymmetries. The project is to determine the cause of the L-G relation. We admit up front that we have failed to do this definitively. But as a contribution to the project, we have eliminated what seem to be promising possibilities. These fail for different reasons. We suggest also a possibility that we are unable to confirm or eliminate. A valid cause of the L-G relation must explain three things: 1. the linear relation between $\delta_{DD}$ and Dst, 2. the specific proportionality factor in this relation, and 3. its dawn-dusk alignment. We approach the search for a cause by translating the asymmetry, $\delta_{DD}$, into a physical force.

5. Can the Pressure of Ring Current Particles Cause the L-G Asymmetry?

[17] Any local-time asymmetry in the low-latitude $H$ component of the magnetic field exerts a force on the Earth equal to the force that the gradient of $H$ through the Earth exerts on Earth’s magnetic dipole. Here we are invoking the rule from electricity and magnetism that the force that an external field exerts on a dipole equals the gradient of the external field times the dipole moment. In the case of the geomagnetic dipole, the rule applies even in the presence of induced currents in Earth’s crust that shield the dipole from external fields [Vasyliunas, 2007]. In this case a force identical to that just stated arises from the shielding currents interacting with the geomagnetic field. Siscoe [1966] applies
the rule to explain how the force that the solar wind exerts on the magnetosphere is transferred to the Earth. The Chapman-Ferraro current that flows on the magnetopause provides the J in the J × B force that 1. stops the solar wind and 2. generates a magnetic field within the magnetosphere having a gradient just right to transfer the force to the Earth. It is necessary that the force be transferred to the Earth for if instead the force were exerted on the resident magnetospheric mass, the mass would be blown out the tail in only a few minutes, which does not happen. The asymmetry in H required to balance the solar wind force is typically between one and two nT/R_E, though at times of strong solar wind it can exceed 10 nT/R_E, as seen in Figure 6. To have a reference number, a gradient of 1 nT/R_E implies a force of 1.3 × 10^7 N.

If we represent the mass of the magnetosphere by the mass of a 10 R_E sphere filled with an H^+ plasma with a density of 10/cm^3 (=17 Kg), the reference force would empty the sphere in less than twenty seconds.

The relevant point here is that the L-G asymmetry implies that there must be a force on whatever it is that generates δ_DD in order to balance the force that δ_DD exerts on the Earth. We will call this the L-G force. In the Chapman-Ferraro example just given, the balancing force is exerted on the solar wind, causing it to stop and flow around the magnetosphere. Our problem is to find something to balance the L-G force. The L-G asymmetry is 20% of Dst, which even in the case of a small magnetic storm (Dst < -50 nT, δ_DD > 10 nT/R_E) gives a L-G force much too strong for any ring current mass to withstand. Consider next the solar wind.

6. Can the Solar Wind Force Cause the L-G Asymmetry?

Note first that the L-G force is directed from dawn to dusk, that is, perpendicular to the solar wind flow. This means that for the solar wind to be responsible for δ_DD, it must push “sideways” (via its static pressure) harder on the dawn flank than on the dusk flank of the magnetopause. It is possible for the solar wind to exert a dawn-to-dusk force if the magnetosphere is shaped like a wing to give “lift” in the duskward direction. For example, if the dusk flank of the magnetosphere is more rounded than the dawn flank (like the top surface of an airplane wing), a duskward lift will result. Such an asymmetry of magnetopause shape might result from ring current pressure being stronger on the dusk side. This possibility also provides the requisite proportionality between δ_DD and Dst, thus satisfying criterion 1 for a viable explanation. An asymmetry of the required sense has, in fact, been reported. The average density on the dawnside flank of the magnetopause is measured to be about 10% greater than on the dusk flank [Paularina et al., 2001]. But so small an asymmetry in pressure gives a net duskward lift too weak to balance the force that δ_DD exerts on the Earth. This is because on the flanks of the magnetopause, the static pressure of the solar wind is about the same as in the free solar wind, namely about 10% of the dynamic pressure. Therefore, a dawn-dusk asymmetry of 10% of the static pressure is about 1% of the dynamic pressure. Applying this ratio to our example, a dawn-dusk asymmetry of 10 nT/R_E would require a noon-midnight asymmetry of around 1000 nT/R_E, which is never observed. It seems safe to eliminate any scenario based on solar wind pressure as a cause of the L-G relation. It violates criteria 2 and 3 for a viable explanation, namely, the proportionality factor is too small and the alignment is more noon-midnight than dawn-dusk.

7. Can a Force on the Plasmasphere Cause the L-G Asymmetry?

The plasmasphere is heavy enough to sustain the L-G force, and because on average the plasmasphere extends farther out into the magnetosphere on the dusk side of Earth than on the dawn side [Carpenter, 1966], a net dawn-dusk
force on the surface of the plasmasphere might conceivably arise to account for the L-G relation. Without trying to invent a mechanism to generate such a force, we can dismiss a plasmasphere solution by estimating the force available to be tapped, namely force \( F \) = pressure \( p \) \( \times \) area \( A \) where \( A \) is the area of the plasmasphere projected onto the \( xz \)-plane and \( p \) is the pressure at the plasmapause. Taking the area to be \( 50 \, \text{R}_E^2 \) and the density at the plasmapause to be \( 300 \, \text{cm}^{-3} \) with a temperature of \( 10^5 \, \text{eV} \) (a high value [Comfort et al., 1988]), gives \( F \sim 10^6 \, \text{N} \), which corresponds to a too-small \( \delta_{DD} \sim 0.1 \, \text{nT}\text{RE}^{-1} \).

[21] Thus, as the source of the L-G force we have eliminated at this point the solar wind, the magnetosphere, and the plasmasphere. We are left with the ionosphere.

8. Can a Force on the Ionosphere Cause the L-G Asymmetry?

[22] Through collisional coupling to the thermosphere, the ionosphere has sufficient effective mass to balance the L-G force. There is already a model of this kind in which the region 1 current system exerts an anti-sunward force on the ionosphere, most of which is balanced by a sunward force on the Earth caused by the gradient in the magnetic field that the region 1 current system generates [Siscoe, 2006; Vasyliunas, 2007]. The associated noon-midnight asymmetry in the disturbance field is seen in Figure 6 (right). A smaller part of the sunward force that balances the region 1 force on the ionosphere is exerted on the solar wind where the region 1 current closes through the magnetosheath.

[23] The region 2 current system generates an asymmetric, low-latitude disturbance field that is commensurate with \( \delta_{DD} \) in terms of strength and in terms of orientation. Thus, the orientation criterion for a viable explanation is satisfied, and the proportionality-constant criterion has the potential to be satisfied since the strength of the region 2 disturbance field is of the right order of magnitude. It remains to identify a physical mechanism that connects the region 2 current system to \( \text{Dst} \) and to derive from it the L-G relation.

[24] To this end we apply three strategic principles:

[25] 1. Look for a mechanism rooted in magnetospheric convection, which is the one global, dynamical process that \( \text{Dst} \) and \( \delta_{DD} \) share.

[26] 2. Construct an explanation from published ideas before looking for new ones, since at this stage in the development of magnetospheric science the discovery of a genuinely novel principle seems unlikely.

[27] 3. Try simple ideas first and add complications as needed, which is using Occam’s razor to guide concept-construction (instead of using it in the usual way to decide between concepts already proposed).

[28] To justify principle 1, we note that magnetospheric convection is considered the prime paradigm for magnetospheric dynamics [Kennel, 1995]. It was conceived already in 1961 as the basic principle that could explain the solar alignment of polar ionospheric current systems, thus connecting the ionosphere through the magnetosphere to the solar wind [Dungey, 1961; Asford and Hines, 1961]. Fairfield and Cahill [1966] confirmed that the interplanetary magnetic field (IMF) is the agent coupling the solar wind to the magnetosphere and ionosphere, as Dungey [1961] had proposed, invoking the concept of magnetic reconnection.

Akasofu and Chapman [1964] speculated that the cause of asymmetry in the low-latitude disturbance field is a partial ring current closing through the ionosphere via field-aligned currents. Cummings [1966] modeled the partial ring current with an actual, physical electrical circuit and showed that the concept can reproduce the observed asymmetry. Crooker and Siscoe [1971] showed that the asymmetry results mainly from the field-aligned part of the partial ring current circuit. They subsequently related the asymmetry magnitude, \( \delta_{DD} \), to the asymmetry-causing field-aligned current, \( I_B \), (also called Birkeland currents) and to the transpolar potential, \( \Phi \), which measures the rate of magnetospheric convection at ionospheric altitude [Crooker and Siscoe, 1981].

\[
d_{DD} = (\mu_0/4\pi\text{RE})I_B \cos \lambda = (\mu_0/4\pi\text{RE}) \Sigma_H \Phi \cos \lambda \approx 0.017 \delta_{DD}(\text{kV})
\]  

(2)

Here \( \Sigma_H \) is the auroral zone Hall conductance, and \( \lambda \) is the latitude at which the field-aligned currents enter and leave the ionosphere. In the last expression, \( \delta_{DD} \) is in nT/RE and we take \( \lambda = 70^\circ \). Equation (2) is based on a steady state version of the Vasyliunas [1970, 1972] magnetosphere-ionosphere coupling model in which there is perfect shielding between auroral and sub-auroral ionospheres. In this idealized case, there results an excess of current flowing into the ionosphere at noon and out at midnight (see Figure 6, left). The first equality in (2) connecting \( \delta_{DD} \) to \( I_B \) is obtained from the configuration of radial field-aligned currents as shown in Figure 6 (right). It does not depend on a steady state assumption. The second equality in (2) connecting \( I_B \) to \( \Phi \) does invoke a steady state assumption.

[29] Equation (2) represents the simplest relation between \( \delta_{DD} \) and a measure of magnetospheric convection, \( \Phi \). The next step in applying strategic principle 1 is to find a corresponding equation relating \( \text{Dst} \) and \( \Phi \). Necessarily it will have been developed to address a different problem than that for which (2) was developed. From prior works (principle 2) the equation that fits the purpose is the now-famous Burton-McPherron-Russell (BMR) equation [Burton et al., 1975].

\[
d\text{Dst}/dt = -aV_B_s - \text{Dst}/\tau
\]  

(3)

where \( V \) is solar wind speed, \( B_s \) is the southward component of the IMF (zero if northward), and \( \tau \) is an empirically determined decay time. In the Burton et al. paper the coefficient \( a \) was determined empirically to be 5.4 nT/mV/m-hr, and the decay time was found to be 7.7 h. O’Brien and McPherron [2000] subsequently re-evaluated these quantities, but at this proof-of-concept stage, we are at the Burton et al. level of analysis and for the present adopt their values. The quantity \( V_B_s \) is the y-component in the GSM coordinate system of the motional electric field in the solar wind. It can be replaced by \( \Phi \) through any of several empirically determined relations as listed by Reiff and Luhmann [1986], the simplest (principle 3) being

\[
V_B_s(nV/m) \approx \Phi(kV)/50
\]  

(4)

Then the BMR equation becomes

\[
d\text{Dst}/dt(nT/hr) = -a'(nT/kV - hr) \Phi(kV) - \text{Dst}(nT)/\tau(hr)
\]  

(5)
The coefficient $a'$ in these units is 0.11 nT/kV-hr. Note that the relation between $Dst$ and $\Phi$ in (5) is explicitly time-dependent, whereas the relation between $\delta_{DD}$ and $\Phi$ in (2) is explicitly time-independent; thus an incompatibility between $Dst$ and $\delta_{DD}$ already arises when they are first brought together through their relations to magnetospheric convection. Nonetheless, at this point we can perform a rough reality check to see whether the numbers come near agreeing. We can make (3) time independent by assuming that the process has reached steady state, although this probably never happens in a storm. Then we can eliminate $\Phi$ between (2) and (5) and use the stated values for $a'$ and $\tau$ to arrive at

$$\delta_{DD}(nT/R_E) = -0.02 \Sigma_H Dst(nT)$$

(6)

We recover the L-G relation if $\Sigma_H = 10$ S, which is not an unreasonable value for ionospheric conduction in the auroral zone. Thus if $\Sigma_H = 10$ S, a $\Phi$ that in steady state gives a Burton et al. value of $Dst$ also gives a value for $\delta_{DD}$ by the field-aligned-current equation (2) that satisfies the L-G relation. But the salient characteristic of magnetic storms is time dependence, which violates the condition under which (6) holds. Indeed, if one solves (5) for $Dst$ using a step function $\Phi$ (0 to 100 kV for 10 h then to 0, say) to simulate an ideal storm, at no time during the storm is the L-G relation satisfied with $\delta_{DD}$ given by the steady state equation (2). Thus, the problem is to find a time-dependent, BMR-like replacement for (2), and to do so from existing literature (principle 2).

9. A Time-Dependent Equation for $\delta_{DD}$

The first equality in equation (2), which relates $\delta_{DD}$ to field-aligned currents, $I_B$, is valid for time-dependent situations, as already noted. Time-dependent models for field-aligned currents are byproducts of efforts to derive analytical formulas to follow the motion of the plasma sheet from the tail into the magnetosphere under a convection potential imposed as an initial condition on a polar cap boundary in the ionosphere [Vasyliunas, 1972; Jaggi and Wolf, 1973; Southwood, 1977; Siscoe, 1982]. Under an imposed convection potential, charge separation occurs at the inner edge of the plasma sheet, a counter-convection electric field results and builds as the motion proceeds until it cancels the imposed potential, and earthward motion stops (an instance of Lenz’s law). The cited references formulate this process in different ways (and describe it in different words) to derive equations that relate the stopping distance of the plasma sheet to the amplitude of the imposed potential. The resulting formulas agree with one another reasonably well, despite being reached from different approaches (although Vasyliunas shows that the answer is not unique).

Among the cited literature on plasma sheet convection, one finds an equation for $\delta_{DD}$ that has the structure of the BMR equation (5) [Siscoe, 1982]. It is based on the following argument. (Invoking the simple-things-first principle 3, we are not distinguishing here between plasma sheet and ring current.) The convection potential that is applied around the polar cap boundary in the ionosphere spreads an electric field through the ionosphere. Equatorward of the polar cap, this electric field maps along equipotential magnetic field lines to the plasma sheet where it induces sunward motion. As argued in Siscoe [1982], the energy required to move the plasma sheet earthward against increasing magnetic field strength comes from the Poynting flux of the region 1 current system because the region 1 current system connects the “driving” potential (applied in the ionosphere) to the solar wind flow, which is the source of the energy. The Poynting flux of the region 2 current system then takes the energy from the region 1 system to the plasma sheet after it passes through the ionosphere across the auroral oval.

The operative electrical circuit is sketched in Figure 7. There are three circuit elements: a battery representing the solar wind as a power supply and resistors, $R_A$, representing ionospheric resistance across the auroral oval. The third element is an inductor, $L$, representing the plasma sheet. The plasma sheet functions as an inductor in this circuit because it is an energy storage element with the property that in steady state it carries current (therefore it is not a capacitor) and dissipates no energy (therefore it is not a resistor). The circuit illustrated in Figure 7 omits ionospheric currents that connect input and output terminals of the region 1 current system by flowing across the polar cap or along the auroral zone or at sub-auroral latitudes. These omissions are made in part in the spirit of principle 3 (make things as simple as possible) and in part because they merely remove flow channels that carry less current than the region 2 channel as indicated by total region 2 current being very close to total region 1 current during disturbed times [Iijima and Potemra, 1978]; thus, alternative closure routes are less important. The circuit equation is

$$\Phi = I_B/\Sigma_P + L dI_B/dt$$

(7)

where $\Sigma_P$ is Pedersen conductance. From equation (2) with $\lambda = 70^\circ$

$$I_B(\text{MA}) = \delta_{DD}(nT/R_E)/17$$

(8)
Figure 8. Plots of $\delta_{DD}$ versus $Dst$ calculated from equations (10) and (11) with $\Sigma_P = 10$ S and varying $\tau_s$ as shown. The curves converge on the Love-Gannon relation, which holds for $\tau_s = 7.7$ h.

(7) may then be rewritten as

$$d\phi_{DD}/dt(nT/R_E/hr) = 0.017 \Sigma_P \Phi(kV)/\tau_s(hr) - \delta_{DD}/\tau_s(hr) \quad (9)$$

where $\tau_s = \Sigma_P L$ is the characteristic time required for the counter-convection electric field, mentioned above, to become comparable to the applied electric field. This is referred to as shielding time.

[35] Now we have time-dependent equations for $\delta_{DD}$ and $Dst$ that are formally similar. In consistent units, these are

$$d\delta_{DD}/dt = 0.017 \Sigma_P \Phi/\tau_s - \delta_{DD}/\tau_s \quad (10)$$

$$dDst/dt = -0.11 \Phi - Dst/\tau \quad (11)$$

Equations (10) and (11) bring the search for commensurate equations for $Dst$ and $\delta_{DD}$ to a successful conclusion. We note again in passing that the original BMR relation (3) has been superseded by empirical relations in which the coefficient $a$ and the decay time $\tau$ are functions of $\Phi$ [O'Brien and McPherron, 2000] and in which the driving term is quadratic in $\Phi$ [Siscoe et al., 2005]. But these additions do not change the ability to fit the data enough to override our use here of principle 3 (add complications only if needed).

10. Deriving the Love-Gannon Relation and Extracting Its Meaning

[34] For the Love-Gannon relation to emerge from (10) and (11) requires the coefficient of $\Phi$ in (10) to equal 0.2 times the coefficient of $\Phi$ in (11) and that $\tau_s = \tau$. Then equations (10) and (11) are identical under the Love-Gannon transformation $\delta_{DD} = 0.2 Dst$. The stated requirements give two equations for $\Sigma_P$, $\tau_s$, and $\tau$, which we may solve for $\Sigma_P$ in terms of $\tau$

$$\Sigma_P = 1.29 \tau \quad (12)$$

If we use the value that BMR give for $\tau$ (7.7 h), we find that for equations (10) and (11) to account for the L-G relation it must be the case that (to some degree of approximation) $\Sigma_P = 10$ S and $\tau_s = 7.7$ h. The first requirement, $\Sigma_P = 10$ S, is actually reasonable. Although Pederssen conductance at the latitude of the region 2 currents is generally less than 10 S (according to IRI2007), during disturbed times, which provide most of the data in which the L-G relation expresses itself, $\Sigma_P$ increases. For instance, Wallis and Budzinski [1981] find that for $Kp > 3$, the contours of average $\Sigma_P$ have longitudinally broad peaks above 10 S, and Vickrey et al. [1981] state that during substorms $\Sigma_P$ rises to values exceeding 25 S. Moreover, Crooker and Siscoe [1981] show that the presence of region 2 currents could raise the effective value of $\Sigma_P$ (that is, the value given by $I_1 = \Sigma_P \Phi$) by a factor between 2 and 4. Thus, it is not unlikely that the average, effective, disturbed-time, auroral-zone value of $\Sigma_P$ is around 10 S.

[35] Regarding $\tau_s = 7.7$ h, we have here a more interesting problem if we indeed take $\tau_s$ to be the shielding time scale. The shielding timescale is usually put at less than one hour [Senior and Blanc, 1984]. But there are indications that it might exceed one hour. For example, Peymirat et al. [2000] state that the time-to-equilibrium of disturbances measured at equatorial latitude following a sudden disturbance recorded at high latitudes can be as much as three hours. Moreover, Jaggi and Wolf [1973], in one of the earliest numerical treatments of the magnetosphere-ionosphere coupling problem, show that the shielding time depends on local time because ionospheric conductance depends on local time. For example, their solution for $\tau_s$ gives about three minutes at midnight and about five hours at noon. Although these numbers can be updated using conductance values deemed more fitting to the situation, the point remains that $\tau_s$ is much longer at noon than at midnight. Thus, the appropriate value to use in the lumped circuit representation of Figure 7 might be significantly longer than the nominal 20 min value suggested by Senior and Blanc. Countering this reassessment of $\tau_s$, however, is a reassessment of the ring current decay time, $\tau$, by Weygand and McPherron [2006], which gives a range from seven hours to one day. Hence although it is possible that the values of $\tau_s$ and $\tau$ can be brought into agreement through adjusting the independently determined values of ionospheric conductance and ring current decay mechanism, we find this option hard to implement and to test.

[36] Another, more testable, option to pursue in reconciling the time-scale discrepancy is to say that we have simply misinterpreted the meaning of $\tau_s$ in equation (7). Instead of the shielding time scale, which depends on conductance, it might actually represent the ring current time scale in the BMR equation itself, that is, in fact it might be that $\tau_s$ is not the $\tau_s$ of MI coupling theory, but is, instead, identical to $\tau$. A physical implication would be that the ring current as measured by $Dst$ feeds current into the ionosphere in a circuit that sends current in around noon and out around midnight, as in Figure 6. Then the identity of the two time scales would follow as a natural consequence.

[37] Figure 8 illustrates the importance of the value of $\tau_s$ in reproducing the L-G relation from equations (10) and (11) with $\Sigma_P = 10$ S. It must get very close to the ring current decay time scale (7.7 h) and almost certainly it must be greater than the upper limit of computed and observed shielding time scale. If $\tau_s$ is in the range of the shielding time scale, the curves are much too open to represent the linear L-G relation. The $\tau_s = 7$ h curve has narrowed to an almost linear shape. Of course, at $\tau_s = \tau$ (7.7 h, not shown) the curve is identical with the L-G relation.
[38] A possible test of the \( \tau_s \equiv \tau \) interpretation can be constructed as follows. Equation (7) implies that the energy in the ring current, \( E_r \), can be expressed in terms of the circuit elements as \( 1/2 L f B \). (This is perhaps pushing the electrical circuit analogy too far, but in the spirit of a test, it could reveal a nonsensical clash of numbers.) We also have by the Dessler, Parker, Schopke relation between ring current energy and \( Dst \) [Dessler and Parker, 1959; Schopke, 1966], \( E_r = -1/3 \mu G Dst \), where \( \mu G \) is Earth’s magnetic moment and we have allowed a \( 3/2 \) amplification of \( Dst \) owing to earth currents (i.e., \( Dst = 3/2 \Delta H \)). Equating the two expressions for \( E_r \) and using (7) with \( \tau = \tau_s = \Sigma P L \) gives a prediction for \( Dst \) in terms of \( I_B \).

\[
Dst(nT) = -24 I_B(MA)^2
\]

where we have set \( \mu G = 8 \times 10^{22} \text{Am}^2 \). The expression is independent of \( \Sigma P \). It is important to recall (since this will challenge the ingenuity of observers) that the quantity \( I_B \) that enters (13) is that part of the field-aligned current that produces the dusk-dawn disturbance asymmetry quantified by \( \delta_D \). It is obvious that (13) states a not-impossible relation between \( Dst \) and \( I_B \). To illustrate, take \( I_B = 2 \text{ MA} \), then (13) gives \( Dst \approx -100 \text{ nT} \). These values do not strike one as outside expectation. Thus, as far as available reality checks can be applied, the meaning of the L-G relation that we are suggesting—that the ring current causes \( \delta_D \) through directly “driving” a noon-midnight field-aligned current system—cannot be dismissed immediately.

[39] We regard the arguments in this section leading to the suggestion \( \tau_s \equiv \tau \) to be the paper’s main contribution. Experts in the ring current modeling community are better prepared to see how a ring current driven, field-aligned current as implied by this identity might arise [e.g., Fok et al., 1995; Liemohn et al., 2001; Kozyra and Liemohn, 2003, and references therein]. But we can suggest one possibility. Since \( \tau \) in the BMR relation is the empirically measured ring current decay time, \( \tau_s \) must have something to do with ring current decay. The L-G relation might be telling us that the decay process is asymmetric, that it creates a persistent asymmetry in the otherwise naturally formed ring current in just the way required to cause current to flow from the ring current into the ionosphere on the dayside and out on the nightside. If an asymmetric decay mechanism exists, one might approach the L-G problem directly from it rather than going through the argument given here, that is, the mutual connection of \( \delta_D \) and \( Dst \) to \( \Phi \). In this eventuality, the argument given here can be thought of as a ladder leading to the conclusion \( \tau_s \equiv \tau \) and its implication in terms of ring current processes. The ladder can then be kicked away.

11. Summary

[40] The recently discovered Love-Gannon (L-G) linear relation between \( Dst \) and the low-latitude, dusk-dawn asymmetry in the disturbance field, \( \delta_D \), (equation (1)) reveals a heretofore unsuspected aspect of magnetospheric physics. The mechanism that causes the L-G relation appears to operate essentially all of the time (Figure 1), although it is often overwhelmed by asymmetries from other causes, such as substorms, dynamic pressure of the solar wind, and unshielded region 1 current (Figures 3 and 5). Apparently the presence of these larger asymmetries has allowed the L-G relation to escape notice until recently, despite its being well expressed if \( \delta_D \) is simply plotted against \( Dst \) (Figures 1 and 3). Another contributor to its late discovery is that no one had predicted it, so no one was looking for it. Before the discovery of the L-G relation, the quantities \( \delta_D \) and \( Dst \) were treated as measuring qualitatively different things—namely, the partial ring current and the symmetric ring current—which were thought to progress in time according to separate rules—partial ring current dominating during the main phase, then symmetric ring current dominating during the recovery phase [e.g., Liemohn et al., 2001; Kozyra and Liemohn, 2003]. There was no reason to suspect that they developed together, one a fixed fraction of the other regardless of storm phase.

[41] To repeat, the L-G relation replaces an expected tandem sequence of \( \delta_D \) and \( Dst \) with simultaneity; and herein lies the problem. The L-G relation does not fit the idea that the partial ring current and the symmetric ring current are different manifestations of magnetospheric convection as measured by the transpolar potential. A different concept appears to be needed. This paper explored ways of generating \( \delta_D \) with solar wind lift, magnetospheric currents other than the partial ring current, and currents on the plasmasphere. These were shown to be unable to provide sufficient pressure or mass to balance the force on the Earth that the gradient \( \delta_D \) induces (Sections 4, 5, and 6). Thus, a partial ring current closing through the ionosphere seems to be required. The next step was to consider whether the partial ring current responsible for generating the Love-Gannon asymmetry might not be the same partial ring current that is studied in connection with region 2 currents and the associated shielding phenomenon. This possibility looks unpromising because the time scales for the two phenomena differ significantly unless special adjustments are made to ionospheric conductance and ring current decay time. An alternative possibility is that instead of being produced by the partial ring current associated with the standard M-I coupling theory, \( \delta_D \) might be produced by a partial ring current associated with the mechanism responsible for ring current decay.

[42] In brief, the L-G relation says that statistically the asymmetry \( \delta_D \) is linearly related to \( Dst \). \( Dst \) has a fairly well determined characteristic decay time. For linearity to hold, therefore, \( \delta_D \) must have a comparable decay time. Moreover, \( \delta_D \) must arise from a current that flows from the ring current to the ionosphere in order that the intimate connection between \( \delta_D \) and \( Dst \) be maintained and in order to provide mass enough to balance the force that \( \delta_D \) exerts on the geomagnetic dipole. The simplest way to meet these requirements in which the ring current decay time plays such a crucial role is for the decay mechanism to cause \( \delta_D \) by setting up a partial ring current that closes through the ionosphere.

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